RELATION BETWEEN COEFFICIENTS OF HEAT CONDUCTION AND DYNAMIC VISCOSITY OF FLUIDS

A. M. Mamedov

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A relation is established between the coefficients of heat conduction and of dynamic viscosity of liquids by using the available experimental results.

In [1-4] it was established experimentally by us that at a given temperature the coefficients of heat conduction and of dynamic viscosity of five aromatic liquid hydrocarbons (benzene, toluene, o-m-p acids) and water are expressed by equations similar to those of the equation of state, namely:

for aromatic hydrocarbons

$$pv/RT = 1 + B\rho + H\rho^7, \quad (1)^*$$

$$\lambda/\lambda_s' = 1 + B_\lambda\rho + H_\lambda\rho^7, \quad (2)$$

$$\eta/\eta_s' = 1 + B_\eta\rho + H_\eta\rho^7; \quad (3)$$

for water

$$pv/RT = 1 + B\rho + E\rho^4, \quad (4)$$

$$\lambda/\lambda_s' = 1 + B_\lambda\rho + E_\lambda\rho^4, \quad (5)$$

$$\eta/\eta_s' = 1 + B_\eta\rho + E_\eta\rho^4. \quad (6)$$

For $\lambda = \lambda_s'$, $\eta = \eta_s'$ one obtains from Eqs. (2), (3) and (5), (6) the following:

for aromatic hydrocarbons

$$-\left(\frac{B}{H}\right)_\lambda = -\left(\frac{B}{H}\right)_\eta = \rho_s^6, \quad (7)$$

for water

$$-\left(\frac{B}{H}\right)_\lambda = -\left(\frac{B}{H}\right)_\eta = \rho_s^3. \quad (8)$$

The validity of the relations (1)-(3) and (4)-(6) is confirmed by the linear dependence of the complexes $[(pv/RT) - 1]/\rho$, $[(\lambda/\lambda_s') - 1]/\rho$, and $[(\eta/\eta_s') - 1]/\rho$ on ρ^6 for aromatic hydrocarbons:

$$\left(\frac{pv}{RT} - 1\right)/\rho = B + H\rho^6, \quad (1')$$

*This brings to mind the particular case of the equation of state for real gases in the virial form which includes the second and the eighth virial coefficient.

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$$\left(\frac{\lambda}{\lambda_s'} - 1\right)/\rho = B_\lambda + H_\lambda \rho^6, \quad (2')$$

$$\left(\frac{\eta}{\eta_s'} - 1\right)/\rho = B_\eta + H_\eta \rho^6, \quad (3')$$

and on ρ^3 for water:

$$\left(\frac{pv}{RT} - 1\right)/\rho = B + E\rho^3, \quad (4')$$

$$\left(\frac{\lambda}{\lambda_s'} - 1\right)/\rho = B_\lambda + E_\lambda \rho^3, \quad (5')$$

$$\left(\frac{\eta}{\eta_s'} - 1\right)/\rho = B_\eta + E_\eta \rho^3. \quad (6')$$

The numerical values of the coefficients in Eqs. (1)-(3) and (4)-(6) as well as a procedure for their determination can be found in [1-4].

The above-stated complexes are linear, namely, the linearity of compressibility, heat conduction, and viscosity for toluene is shown in Fig. 1a,b,c.

It should be mentioned here that the indices 7 and 4 in Eqs. (1)-(3) and (4)-(6) were established entirely empirically by us. The tests have shown that if for toluene the index 7 is replaced by 6 or 8 the linearity of the complex suffers, as one can see from Fig. 2a,b.

The construction of equations of motion was studied by Putilov who, in [5], proposed the equation of state for fluids

$$p + A/v^\mu = RT/v + B/v^\nu, \quad (9)$$

starting from the equilibrium of four pressures, where p is the external pressure; RT/v is the thermal pressure; A/v^μ is the attraction pressure due to molecular attraction; B/v^ν is the repulsion pressure due to molecular repulsion.

The index ν in Eq. (9) is taken as equal to 4 and it is proposed to evaluate μ with the aid of the Young criterion,

$$\mu = \frac{3J_0}{3J_0 - 4}, \text{where } J_0 = RT_{cr}/p_{cr}v_{cr}.$$

The transformed equation (9) results in

$$\left(\frac{pv}{RT} - 1\right) v^{\mu-1} = -A^* + B^*/v^{\nu-\mu}, \quad (9')$$

and the fluid isotherms in the coordinate system

$$\left(\frac{pv}{RT} - 1\right) v^{\mu-1} \text{ and } 1/v^{\nu-\mu}$$

should be represented by straight lines.

It is not difficult to see that Eqs. (1) and (2) introduced by us are particular cases of Putilov's equation (9).

By applying Eq. (9) to the fluids nitrogen and benzol, for which experimental values of p, v, and T are available in [6, 7], it was shown that the isotherms of such fluids in these coordinate systems are not rectilinear.

The above indicates that the values of the indices μ and ν obtained by the above-described procedure do not ensure that Eq. (9) can be applied to other fluids.

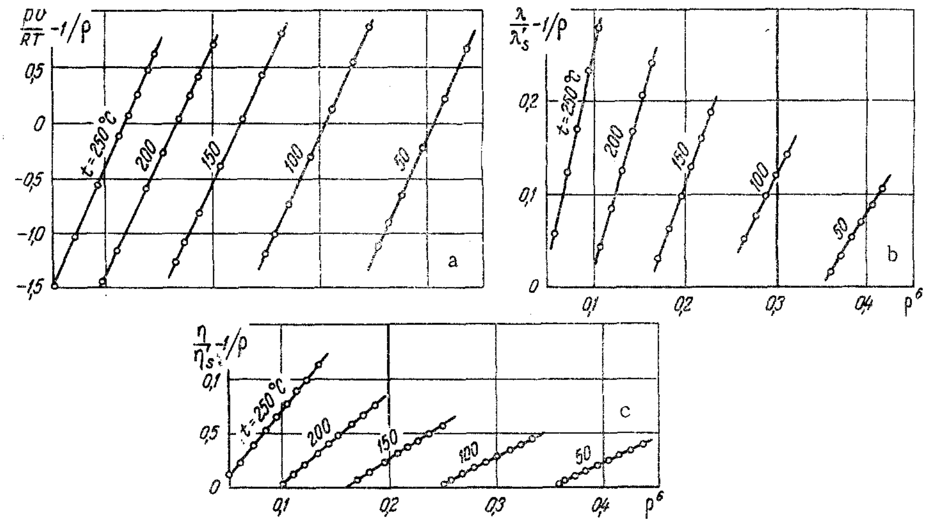

Fig. 1. The linearity of complexes of (a) compressibility, (b) heat conduction, and (c) viscosity for toluene.

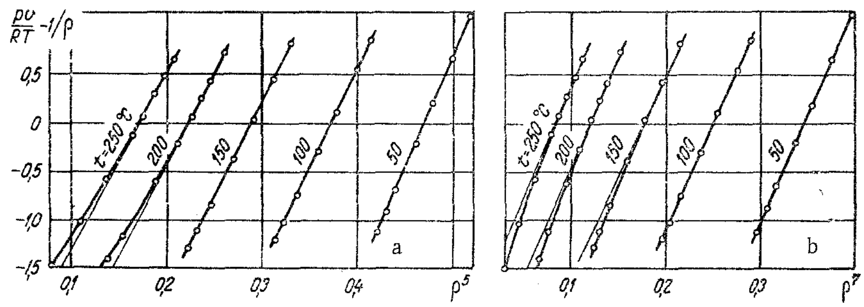

Fig. 2. Infringement of linearity for the compressibility complex with indices 6 (a) and 8 (b).

In view of the fact that there is no approach based on physics to find the values of the indices μ and ν our starting point in [1-4] was the empirical determination for each fluid as already mentioned above.

To find the relation between the coefficients of heat conduction and dynamic viscosity of fluids the formulas proposed by us are again transformed. Namely, by using the relations (7) and (8), Eqs. (2), (5) and (3), (6) are transformed into the following form:

for aromatic hydrocarbons

$$\left(1-\frac{\lambda/\lambda_s'-1}{H_\lambda\rho^7}\right)\left(\frac{\rho}{\rho_s}\right)^6=1, \tag{10}$$

$$\left(1-\frac{\eta/\eta_s'-1}{H_\eta\rho^7}\right)\left(\frac{\rho}{\rho_s}\right)^6=1; \tag{11}$$

for water

$$\left(1-\frac{\lambda/\lambda_s'-1}{E_\lambda\rho^4}\right)\left(\frac{\rho}{\rho_s}\right)^3=1, \tag{12}$$

$$\left(1-\frac{\eta/\eta_s'-1}{E_\eta\rho^4}\right)\left(\frac{\rho}{\rho_s}\right)^3=1. \tag{13}$$

Moreover, by comparing (10) and (11) as well as (12) and (13), one obtained

$$\frac{\lambda/\lambda_s' - 1}{H_\lambda} = \frac{\eta/\eta_s' - 1}{H_\eta}$$

or, by virtue of the relation (7), one has

$$\frac{\lambda/\lambda_s' - 1}{\eta/\eta_s' - 1} \cdot \frac{B_\eta}{B_\lambda} = 1. \tag{14}$$

It is not difficult to see that this equation implies a linear dependence between the coefficient of heat conduction and that of dynamic viscosity of liquids. In [3] this linear dependence was confirmed for water.

For two states with the parameters p_1, t_1 and p_2, t_2, Eq. (14) for the fluid under consideration becomes as follows:

$$\left(\frac{\lambda/\lambda_s' - 1}{\eta/\eta_s' - 1} \cdot \frac{B_\eta}{B_\lambda}\right)_1 = \left(\frac{\lambda/\lambda_s' - 1}{\eta/\eta_s' - 1} \cdot \frac{B_\eta}{B_\lambda}\right)_2 . \tag{15}$$

For two states with the parameters p_1, t and p_2, t, Eq. (15) simplifies to

$$\frac{\lambda_1 - \lambda_s'}{\lambda_2 - \lambda_s'} = \frac{\eta_1 - \eta_s'}{\eta_2 - \eta_s'} . \tag{16}$$

Thus, in accordance with Eq. (15) a unique relation is obtained for some aromatic hydrocarbons as well as for water; the latter provides a basis for representing the relation between transferable properties of fluids, as follows:

$$\frac{\lambda/\lambda_s' - 1}{\eta/\eta_s' - 1} \cdot \frac{B_\eta}{B_\lambda} = \text{idem} \tag{17}$$

or

$$\frac{\lambda - \lambda_s'}{\eta - \eta_s'} \cdot \left(\frac{\eta_s'}{\lambda_s'} \cdot \frac{B_\eta}{B_\lambda}\right) = \text{idem}, \tag{18}$$

where

$$\frac{\eta_s'}{\lambda_s'} \cdot \frac{B_\eta}{B_\lambda} = f(t)$$

or by using (7) or (8)

$$\frac{\eta_s'}{\lambda_s'} \cdot \frac{H_\eta}{H_\lambda} = f(t). \tag{19}$$

It should be mentioned in conclusion that the derivation of (17) was based on Eqs. (2) and (3) as well as (5) and (6), whose form resembles the equations of state (1) and (4). The basic relations for deducing the relation (17) are thus equation of state; the latter include another two forces in addition to the external pressure and thermal pressure, namely, attraction and repulsion due to interactions between the molecules of the fluid [1], or, in other words, between two so-called virial coefficients B and H or B and E.

Since Eqs. (1)-(3) and (4)-(6) have only been verified on a limited number of fluids, the relation (17) obtained from the former should be verified on other fluids.

NOTATION

p is the pressure; T is the absolute temperature; $\lambda(p, T)$ is the coefficient of heat conduction; $\eta(p, T)$ is the coefficient of dynamic viscosity; $\rho(p, T)$ is the density; $\lambda_s'(T)$, $\eta_s'(T)$, $\rho_s(T)$ are the same parameters for saturated fluids; B, B_λ, B_η; E, E_λ, E_η; H, H_λ, H_η are coefficients dependent on temperature only.

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